JKAU: Eng. Sci., **Vol. 34 No. 2**, pp: 81–93 (2024A.D.)
DOI:10.4197/Eng.34-2.6

Continuous Cash Flow Modelling for Economic Evaluation and Risk Analysis in Mine Planning Using Monte Carlo Simulation

Haitham M. Ahmed, Sefiu O. Adewuyi[2] and Hussin A. M. Ahmed[1]

[1]King Abdulaziz University, Jeddah, Faculty of Engineering, Mining Engineering Department
[2]University of Arizona, Tucson, AZ, College of Engineering, Department of Mining & Geological Engineering

sadewuyi@arizona.edu

Abstract

The Discrete Cash Flow (DCF) model is commonly used to estimate the Net Present Value (NPV) of various projects including mining projects. However, for projects with high uncertainties and extended timeframes, such as mining projects, the DCF model may produce results that deviate from actual outcomes. To address this limitation, this paper introduces a Continuous Cash Flow (CCF) model designed specifically for the economic evaluation of mining projects. The model is applied herein to assess the economic feasibility of a gold mining project in Saudi Arabia. For comparative analysis, both the proposed CCF model and the conventional DCF model were used to evaluate the same project, with results obtained through 10,000 iterations using @Risk and simulated via the Monte Carlo technique. Findings indicate that the CCF model yields lower NPV values than the DCF model, with a correlation coefficient of 0.99 between the two approaches. The CCF model thus offers a viable alternative for determining expected NPV in mining projects with significant uncertainties.

Keywords: mine planning, mine valuation, mining economics, mining phases, net present value (NPV), continuous cash flow.

1. Introduction

The decision to invest in any project is a challenge, it becomes particularly complex due to numerous uncertainties. These include variability in orebody geometry and accessibility, land ownership issues, political considerations, lengthy development and construction timelines, skilled labor shortages, worker safety concerns, environmental impacts from excavation and processing, production risks, and price volatility. These factors collectively add significant complexity to investment decisions in the mining sector [1–3].

The profitability or viability of mining projects could be evaluated using several economic indices, including the Net Present Value (NPV), Internal Rate of Return (IRR), Payback Period, Present Value Ratio (PVR) or Profitability Index (PI) [4].

Of these, NPV is the most widely employed due to its ability to provide robust and reliable results, offering a comprehensive measure of the financial performance and potential returns of mining investments. [5]. NPV represents today's value of expected future cash flows over the project's lifespan at a given discount rate. It can be estimated using either the Discrete Cash Flow (DCF) or Continuous Cash Flow (CCF) approach. In the DCF approach, cashes are considered at specified interval, such as weekly, monthly or annually. In contrast, the CCF assumes cash flows occur continuously over the project's timeframe, providing a potentially more precise representation of cash movement. NPV estimated using the DCF approach is calculated as the algebraic sum of the DCF present values, typically consolidated at a specific point on the time axis (usually at t=0). In contrast, the NPV derived from the CCF approach is obtained by integrating continuous cash flow functions over the project's duration and moving the result to the start time (t=0) [6–9].

The NPV estimated using the DCF or CCF approach can be classified as static or dynamic. The term static refers to cases where input parameters have fixed, deterministic values, meaning each parameter is represented by a single, unchanging value. Conversely, dynamic refers to models that incorporate uncertainty in input parameters, allowing for variability that accounts for some of the inherent risks in mining projects[10]. Among the input parameters in economic evaluation of mining projects are capital costs, operational costs, ore reserve, ore grade, mining grade, metal grade, revenue, metal price, interest rate, royalty, and exchange rate [5]. These parameters have varying degrees of uncertainties that need to be accounted for in the economic evaluation of mining projects. The static method does not account for uncertainty in mining projects. Consequently, most mining companies use high discount rates to compensate for uncertainties in the input parameters of mining projects [11]. This can result in the undervaluation of long-term projects, particularly those extending over several decades, and may even render mining projects appear unfeasible.

To compensate for the uncertainties in mining projects, Dehghani and Ataee-pour [12] proposed that metal price and operating cost can be incorporated into the DCF. Ugwuegbu [13] proposed a dynamic DCF model that considered several uncertainties in capital costs, operating costs, gold price, and ore reserve. Many researchers considered different uncertainties in input parameters such as risk in interest rate [14], risk associated with the effect of weather on mining projects [15] and risk in metal price [5]. However, a concern remains that existing models often overlook key uncertainty factors in mining projects, such as annual working days, ore production capacity, dilution rates, metal recovery, and exchange rate fluctuations. It is therefore recommended that these additional uncertainties be incorporated into the DCF method to enhance accuracy. Nonetheless, given the high degree of uncertainty inherent in mining projects, the NPV estimated using the dynamic DCF method may still fall short of fully capturing real-world outcomes [16].

In this context, the CCF approach to estimating NPV has been suggested as a more robust model, as it better accounts for uncertainty in financial analysis, offering a more realistic assessment of project viability. [6]. Zhang and Wu [17] discussed that an economic evaluation system without the CCF method may be regarded as incomplete, further emphasizing the importance of incorporating continuous cash flow models into the evaluation of mining projects. However, the methodology and implementation of the CCF in financial evaluation is limited due to its complexity. Therefore, there is a need to provide a simpler, more practical approach to estimating NPV using the CCF method in mining projects. This would involve considering a set of uncertainty parameters at a time and comparing the results with those derived from the dynamic DCF method, thereby offering a more accessible and effective tool for financial analysis in the mining sector. This paper aims to develop a dynamic CCF model, apply it to a case study, and compare its results with dynamic DCF results for the same case study. For both dynamic CCF and DCF, the Monte Carlo simulation technique was used to quantify the risk associated with the mining project.

2. Dynamic CCF model for mining project

Considering the first four phases of mining projects: development and construction, start-up, full production, and depletion. During the development and construction phase, cash flows are primarily allocated to building necessary facilities, purchasing and installing equipment, developing access roads, and securing other essential resources for mining operations. These activities typically require significant expenditures, known as capital costs [18]. At the start-up phase, the project normally begins production, which gradually increases until the full operational capacity of the facility is reached. Once this milestone is achieved, the production phase officially begins. Thus, the cash flow begins to increase gradually from the project start phase-up phase, eventually reaching a stable production that can persist for many years. This steady production continues until the depletion of the ore reserve, at which point the project transitions into the closure and reclamation phase, marking the final stage of the project lifecycle. Consequently, it can be said that cash flow follows various patterns from the development and construction phase through to closure, reflecting the evolving nature of the project over time. Figure 1 illustrates typical cash flow patterns generally exhibit in mining projects, highlighting the distinct phases from development and construction to production and eventual closure. In both Figures (1A and 1B), t, subscripts; c, s, f, and d represent time of development and construction, start-up, full production, and depletion phase respectively. Using numbers (1, 2, 3 etc.) along with subscripts c, s, f, and d denotes the start or finish of a specific cash flow pattern within the same phase. These subscripts help to differentiate the various stages or transitions of cash flows within the project lifecycle.

Figure (1A) illustrates a scenario where an equal amount of capital cost is allocated at t_{c1}, t_{c2}, and t_{c3}. In contrast, Figure 1B depicts a scenario where a larger proportion of the capital cost is allocated at t_{c1}, with progressively lower expenditures at t_{c2} and t_{c3}. The location and facilities required for different mining projects may contribute to such variations, potentially generating an unlimited number of different scenarios depending on the specific project requirements and conditions. Uncertainty in mining projects can cause fluctuations in cash flows, as seen during the full production phase (between t_{f1} to t_{f4}) in scenario 2 (Figure 1B). These fluctuations reflect the inherent risks and variable factors that impact project performance over time. Both scenarios (Figure 1A and Figure 1B) can be represented by three distinct types of CCF patterns: an increasing cash flow pattern (e.g from $t = 0$ to $t = t_{c1}$), a uniform cash flow pattern (e.g from $t = t_{c2}$ to $t = t_{c3}$), and a decreasing cash flow pattern (e.g from $t = t_{f2}$ to $t = t_d$ in Figure 1A and from $t = t_{f1}$ to $t = t_{f2}$ in Figure 1B). For instance, if the rate of ore production is reduced during the full production phase or any other stage of the project, revenue generation will decline, assuming other parameters remain constant. As a result, the direction of cash flow will shift negatively, reflecting the reduced income from the project. The reverse is true if the production rate is increased or the metal price rises. In such cases, a positive direction of cash flow will occur, provided the operating income remains positive. The cash flow patterns in Figure 1 can be modelled by breaking them down into different geometric shapes. To represent the cash flow accurately, triangular, rectangular, and trapezoidal shapes were used. These shapes effectively capture the variations in cash flow over different phases of the project.

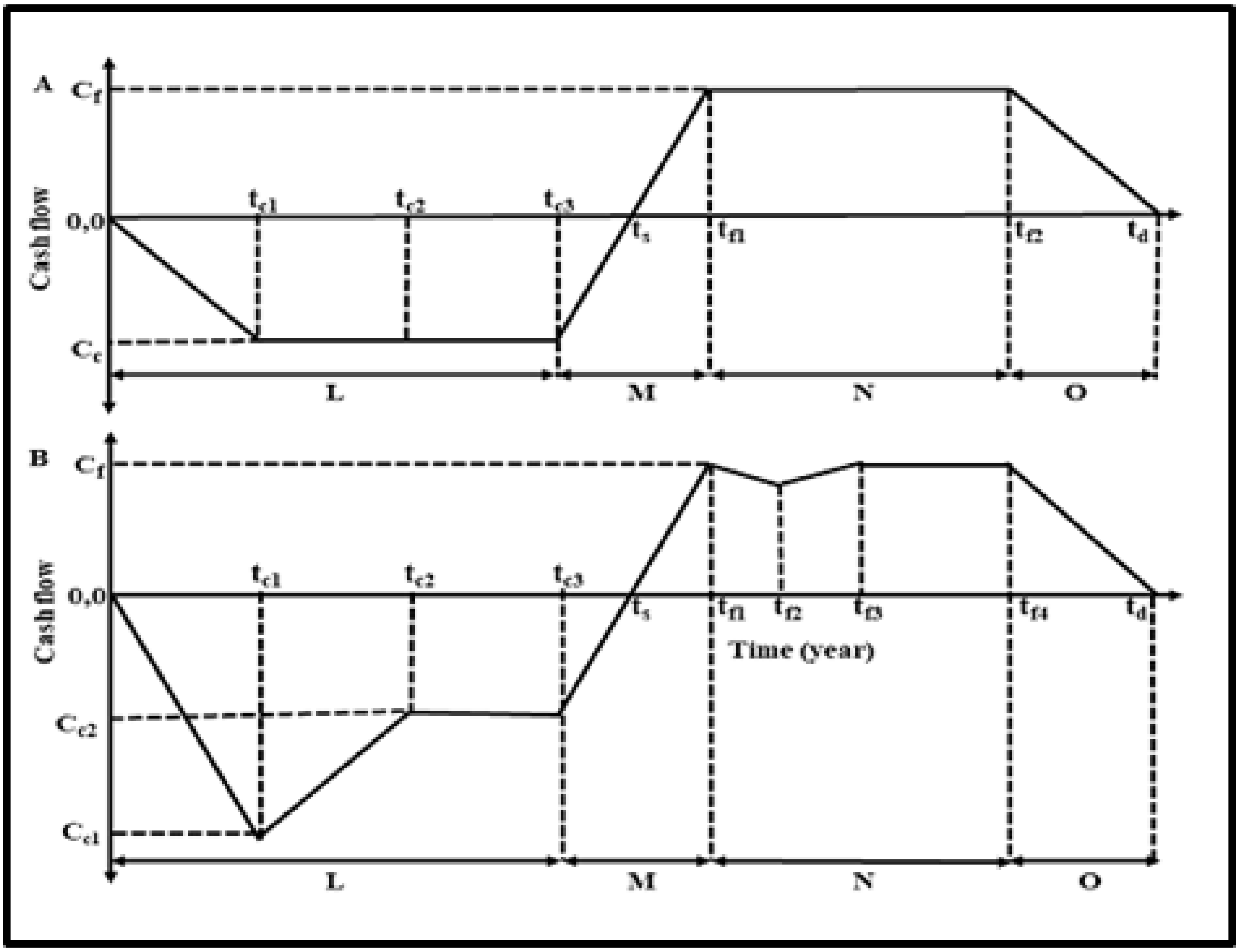

Figure 1. Schematic diagrams of a typical mining cash flow under two different scenarios: (A) gradual developmental and stable production scenario, (B) gradual and reduced capital costs at the developmental phase and fluctuation in production or metal costs. L – Development and Construction phase, M – Start-up phase, N – Full production phase O – Depletion phase

2.1 Modelling continuous cash flow

In the development and construction phase of Figure 1A, the cash flow can be represented by two simplified CCF functions. The first function models the CCF between time t = 0 to $t = t_{c1}$, while the second function covers the interval from $t = t_{c1}$ to $t = t_{c3}$. It should be noted that t_{c1}, t_{c2} and t_{c3} can represent any real units of time (second, minute, hour, day, etc.), depending on the project's specific timeframe and requirements. In the first part of the model, the cash flow rate, $C_{c1}(t)$ decreases from zero to C_c while in the second part, the cash flow rate, $C_{c2}(t)$, is uniform (becomes C_c). The Present Value (PV) for the development and construction phase, PV_c can be obtained as follows:

$$PV_c = PV_{c1} + PV_{c2} \quad (1)$$

where PV_{c1} represents PV for decreasing cash flow rate from $t = 0$ to $t = t_{c1}$, while PV_{c2} represents the PV for the segment of the function where $t = t_{c1}$ to $t = t_{c3}$.

The general representation of the PV of any component of CCF can be obtained using Equation (2).

$$PV = \int_{t_1}^{t_2} C(t)e^{-rt}\, dt \quad (2)$$

where *C(t)* represents the cash flow function at a specific time frame, t (from t_1 to t_2), and r denotes the discount rate.

To determine the PV at $t = 0$, using Equation 2, reformulated specifically for the construction phase depicted in Figure 1A, as shown in Equation (3). In this equation, the first term expresses the PV_{c1}, while the second term represents PV_{c2}, estimated by integrating the function at $t = t_{c1}$ and then continuously discounted back to $= 0$.

$$PV_c = \int_0^{t_{c1}} C_{c1}(t)e^{-rt}\, dt + \left[\int_{t_{c1}}^{t_{c3}} C_{c2}(t)e^{-rt}\, dt\right] e^{-rt_{c1}} \quad (3)$$

In the second scenario (Figure 1B), the cash flow during the development and construction phase consists of three parts: decreasing cash flow from zero to C_{c1}, increasing cash flow from C_{c1} to C_{c2}, and uniform cash flow (C_{c2}) between t_{c2} and t_{c3}. Using the same methodology as applied in scenario 1 (Figure 1 A), the present value, PV_c, for this case can be calculated as shown in Equation 4.

$$PV_c = PV_{c1} + PV_{c2} + PV_{c3} = \int_0^{t_{c1}} C_{c1}(t)e^{-rt}\, dt + \left[\int_{t_{c1}}^{t_{c2}} C_{c2}(t)e^{-rt}\, dt\right] e^{-rt_{c1}} + \left[\int_{t_{c2}}^{t_{c3}} C_{c3}(t)e^{-rt}\, dt\right] e^{-rt_{c2}} \quad (4)$$

where PV_{c1} is the present value for decreasing cash flow from $t = 0$ to $t = t_{c1}$, PV_{c2} is the present value between t_{c1} and t_{c2} and PV_{c3} is the PV between t_{c2} and t_{c3}.

This approach can be extended to model each phase in both scenarios shown in Figure 1. Notably, both scenarios can be represented as combinations of linearly increasing, uniform, and decreasing cash flows. Therefore, the cash flow rate, *C(t)*, in each of the three cash flow forms can be expressed using the general equation of a straight line, as shown in Equation (5).

$$y = mx + k \quad (5)$$

where y = cash flow, m = slope and k = intercept on the cash flow axis.

Equation 6 represents the straight line in Figure 2. The PV for this case is, PV_A (Equation 7), calculated by substituting Equation 6 into Equation 2). Using the same approach, the PV for the case in Figure (2B), PV_B, is as presented in Equation 8.

$$C(t) = C_1 + \left(\frac{C_2 - C_1}{t_1 - 0}\right) t \quad (6)$$

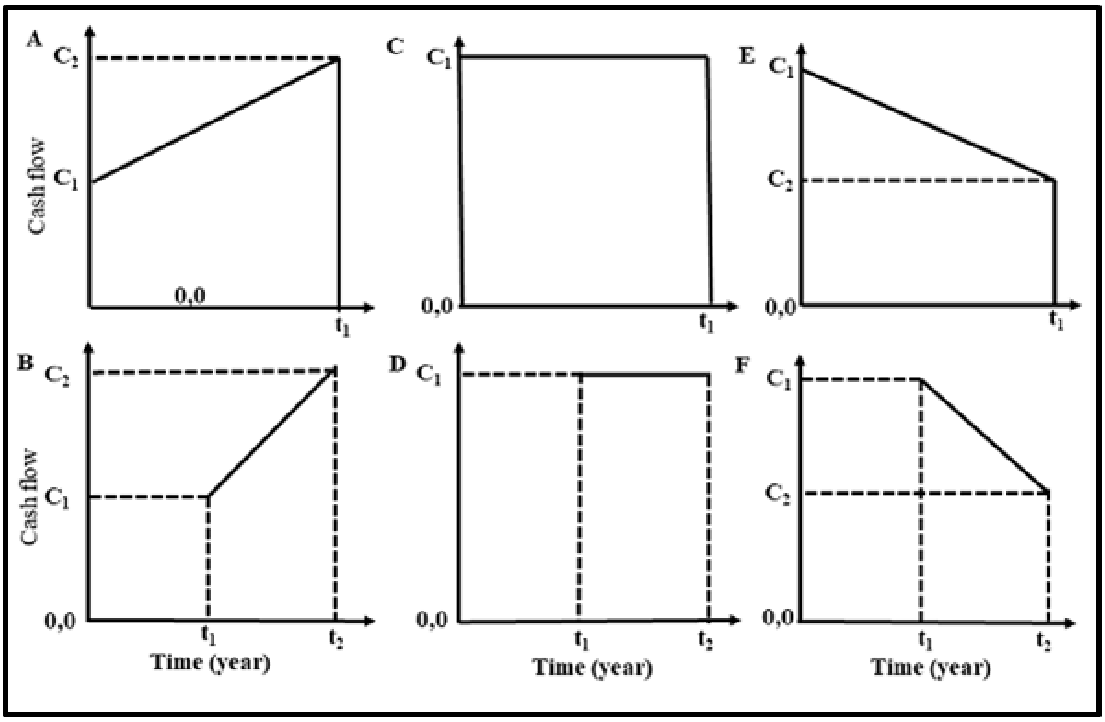

Figure 2. The schematic diagram for different cash flows. A and B – increasing cash flow, C and D – uniform cash flow, and E and F – decreasing cash flow.

$$PV_A = \frac{1}{r}(C_1 - C_2 e^{-rt_1}) + \frac{C_2 - C_1}{t_1 r^2}(1 - e^{-rt_1}) \quad (7)$$

$$PV_B = [\frac{1}{r}(C_1 - C_2 e^{-rt}) + \frac{C_2 - C_1}{tr^2}(1 - e^{-rt})]e^{-rt_1} \quad (8)$$

where $t = t_1 - t_2$

Equation 8 can be used to determine any increasing cash flow at any stage of the mining project, by substituting the appropriate parameters. For uniform cash flow in Figures 2C and 2D, Equations 9a and 9b represent the cash flow functions respectively.

$$C(t) = c_1 \qquad 0 < t < t_1 \quad (9a)$$

$$C(t) = c_1 \qquad t_1 < t < t_2 \quad (9b)$$

The PV of such cash flows can be obtained using Equations 10 and 11.

$$PV_C = \frac{C_1}{r}(1 - e^{-rt_1}) \quad (10)$$

$$PV_D = [\frac{C_1}{r}(1 - e^{-rt})]e^{-rt_1} \quad (11)$$

where PV_C represents the present value of the uniform cash flow in Figure 2C, PV_D represents the present value for the uniform cash flow in Figure 2D, and $t = t_2 - t_1$.

Considering decreasing cash flow in Figure 2E, the Cash flow function can be expressed as presented in Equation 12.

$$C(t) = c_1 - \left(\frac{C_2 - C_1}{t_1 - 0}\right)t \quad (12)$$

The corresponding PV of Equation (12) is NPV_E, (Equation 13).

$$PV_E = \frac{1}{r}(C_2 - C_1 e^{-rt_1}) + \frac{C_1 - C_2}{t_1 r^2}(1 - e^{-rt_1}) \quad (13)$$

where t = (t_2 -t_1).

If the cash flow in Figure 2F is considered then, its PV; PV_F, is as presented in Equation 14.

$$PV_F = [\frac{1}{r}(C_2 - C_1 e^{-rt}) + \frac{C_1 - C_2}{tr^2}(1 - e^{-rt})]e^{-rt_1} \quad (14)$$

where $t = t_2 - t_1$.

Equation (14) can be used to calculate the PV of any decreasing cash flow in the two scenarios in Figure (1).

3.1 NPV of a mining project

Correct input parameters are crucial in getting reliable results of NPV, not only in mining but also in every project. Hence, care must be taken to ascertain the correctness of the input parameters. The NPV of the simplified continuous cash flows is the summation of all individual PV within the project lifespan (Equation 15). If the increasing, uniform, and decreasing models are used to calculate the PV for the development and construction phase – PV_C, start-up phase – PVs, full-production phase – PV_f, and depletion phase – PV_d, then, the NPV of the project can be determined using Equation (15).

$$NPV = PV_c + PV_s + PV_f + PV_d \qquad (15)$$

3 Case study: a gold mining project

Considering a gold mine project in Saudi Arabia that is at the feasibility stage, and the ore reserve has been estimated to be 17.8 million tones. The data in Table 1 was obtained from a pre-feasibility study based on a mining company's consideration of a gold mine project. The company arrived at the base and minimum values of input parameters relying on previous experience with similar projects.

At the start-up phase, production was planned with 25% capacity and increases annually using the same percentage until maximum capacity was reached in the fourth year. At the depletion phase, production was reduced from maximum capacity to 75%, then 50% while the remaining gold ore is to be mined and processed in the last year of production. Based on the above information and data presented in Table 1, the CCF models presented in this paper were used to evaluate the economic viability and risks associated with the project. Equations 16 – 26 were developed to generate the cash flow of the project.

Table 1. A typical gold mine project input parameter.

Input parameters	Symbol	Most likely	Pessimistic	Optimistic	Probability distribution
Capital cost ($)	W	62,000,000	60,000,000	65,000,000	Triangular
Grade (g/t)	G	1.35	1.3	1.35	Lognormal
Price of gold (US $/Oz)	P_G	1072.19	409.72	1668.98	Lognormal
Variable cost ($/t)	V_c	24	22	27	Triangular
Royalty rate	T_R	5.00%	NA	NA	NA
Capacity (t/d)	q	3,400	3,100	3,500	Triangular
Days per year	D	350	340	360	Triangular
Reserve (t)	Q	17,500,000	17,000,000	17,800,000	PERT-Beta
Mining Dilution	M_d	9%	8%	10%	Lognormal
Metal recovery	M_R	90%	85%	95%	Lognormal
Tax rate	Tr	20%	NA	NA	NA
Discount rate	r	9%	8%	10%	Lognormal
Working capital rate	W_r	25%	30%	35%	Triangular
Years of production	T_p	18	NA	NA	NA
Exchange rate	E	3.76	3.75	3.77	Triangular

$$OP = q * D \qquad (16)$$

$$MP = OP * G * M_R * (1 - M_d) \qquad (17)$$

$$R = MP * P_G \qquad (18)$$

$$OC = V_c * OP \qquad (19)$$

$$OI = R - OC \quad (20)$$

$$RO = T_R * R \quad (21)$$

$$CA = W/T_p \quad (22)$$

$$WC = W_r * OC \quad (23)$$

$$TI = OI - RO - CA \quad (24)$$

$$CT = T_r * TI \quad (25)$$

$$CF = OI - W - WC - CA - RO - CT \quad (26)$$

where OP – Ore production, MP – Metal production, R – Revenue, OC – Operating cost, OI –Operating income, RO – Royalty, CA – Capital allowance, WC – Working capital, TI – Taxable income, CT – Corporate tax, and other used terms are as defined in Table 1.

The probability distribution functions applied for each input parameter are presented in Table 1. A triangular distribution was used for capital cost, working capital, operating cost and workday per year while PERT-Beta distribution was employed for ore reserve. The lognormal distribution is commonly employed for the grade, recovery, metal price and dilution factor and therefore used in this study [13]. The mean, μ and standard deviation, σ needed in lognormal and PERT-Beta distribution were calculated using Equation 27 and Equation 28, respectively.

$$\mu = \frac{a+4m+b}{6} \quad (27)$$

$$\sigma = \frac{b-a}{6} \quad (28)$$

where a, m and b are the pessimistic, most likely, and optimistic estimations respectively.

Uncertainty in the price of gold is one of the crucial factors in the risk analysis of any gold mine project. It is highly dependent on the international market and cannot be controlled by the mining company. However, the historical trend of metal prices can be used to forecast future prices during the feasibility study. In this study, historical data on gold prices between 2004 and 2019 was used [19]. The mean and standard deviation were calculated and applied in building the probability distribution for this case study. The exchange rate of Saudi Arabia Riyal (SAR) to the United States of America dollar ($ US) ranged between SAR 3.75 and 3.77 per $ US, since it has been almost stable for the past decade. The tax rate on gold and other metal exploitation in Saudi Arabia is 20% [20]. The obtained NPV was simulated for 10,000 iterations using the Monte Carlo simulation technique implemented in @RISK software, version 7.5.2: Industrial (student) Edition acquired from Palisade software company.

3. DCF model

The same data presented in Table 1 was used to calculate the expected NPV using the conventional dynamic DCF approach. Equation 29 is the DCF model for calculating the NPV of cash flow. To make the model dynamic; input parameters can be varied using the probability density function, as described for the CCF approach. Also, the NPV obtained using the DCF model was simulated using the same iteration number, method and software as in the CCF model. Regression analysis was performed to establish the relationship between the NPV obtained using DCF and CCF models.

$$NPV = \sum_0^T C_i(1 + r)^t \quad (29)$$

where t = 1, 2, 3…. T (the end of the project time).

4. Result and discussion

Figure 3A presents the probability distribution of input parameters presented in Table 1. Figure 4A is the discrete cash flow while Figure 4B is its corresponding continuous cash flow. Figure 5A and Figure 5B present the results of NPV for the dynamic DCF and CCF models, respectively. The comparison of the two methods is presented in Figure 5C while the regression analysis between the dynamic DCF and CCF is presented in Figure 5D. The CCF model produced an NPV of approximately -11.2 M SAR equivalent to

3.0 M SAR in the dynamic DCF model, at 45 % probability. In this case, the CCF model suggests that the project is not likely to be profitable while the dynamic DCF suggests otherwise. At 50 % probability, the CCF model produced an estimated NPV of approximately 5.1 M SAR, equivalent to 20.0 M SAR, using the dynamic DCF method. The results show that the project is highly feasible at 50 % probability. The variation of NPV between the dynamic DCF and CCF is 9 %, at 90 % probability. At 99 % probability, the difference in NPV between the two approaches falls to 6.96 %. It can be observed from Figure 5C that NPV obtained from both CCF and DCF increases as percentage project feasibility increases with a similar pattern. However, the CCF model proposed in this study gives lower NPV values. This could be attributed to the fact that the CCF discounting approach accounts for all possible cash flows within the entire project life, unlike the DCF method where cash flows are assumed to be specific time frame. Results from this study support the earlier opinions of some researchers that the DCF method may overestimate the project profitability [13, 21] and short of real mining projects worth [16]. The regression analysis of NPV calculated using the dynamic CCF and DCF at a 95 % confidence interval shows that the coefficient of determination, r^2 = 0.99 (Figure 5D). A relationship between the dynamic CCF and DCF is presented in Figure 5D which can be used to predict the NPV using the CCF method when that of DCF is known or vice versa.

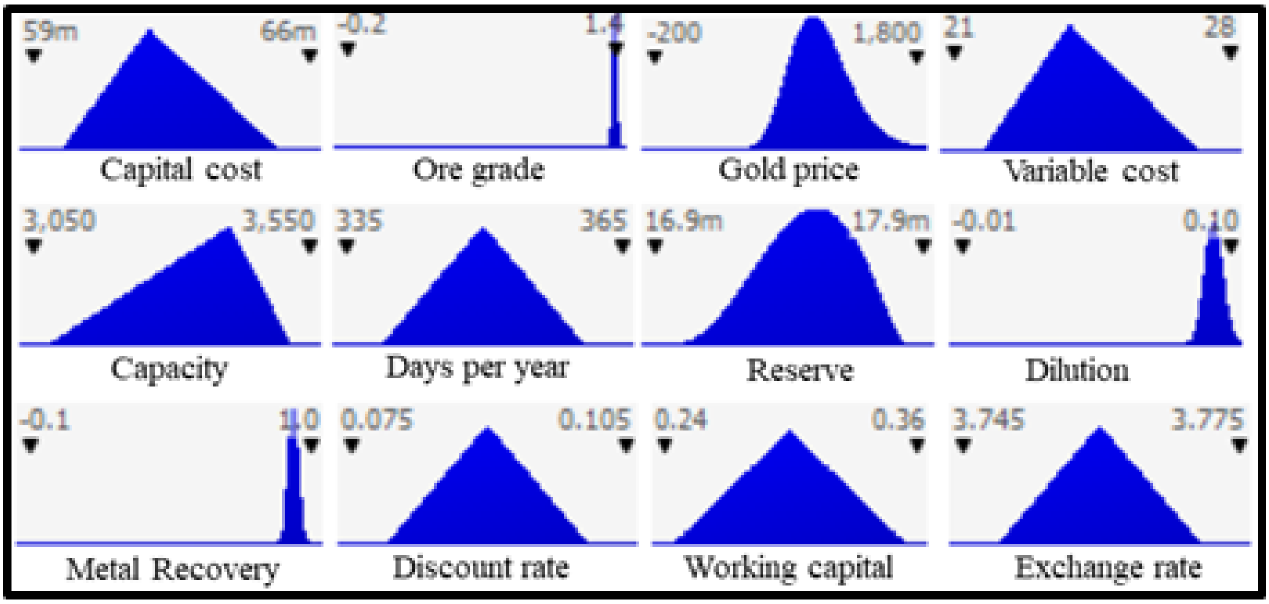

Figure 3. The result of probability distribution of uncertain input parameters estimated after 10000 trials using risk software

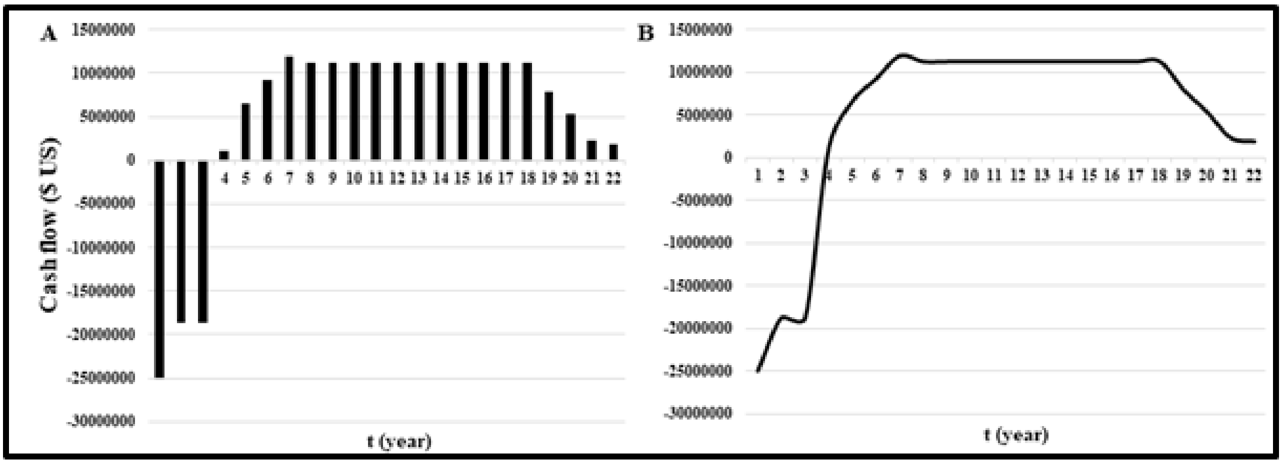

Figure 4. Cash flow results estimated after 10000 trials using @risk software (A) DCF, (B) CCF

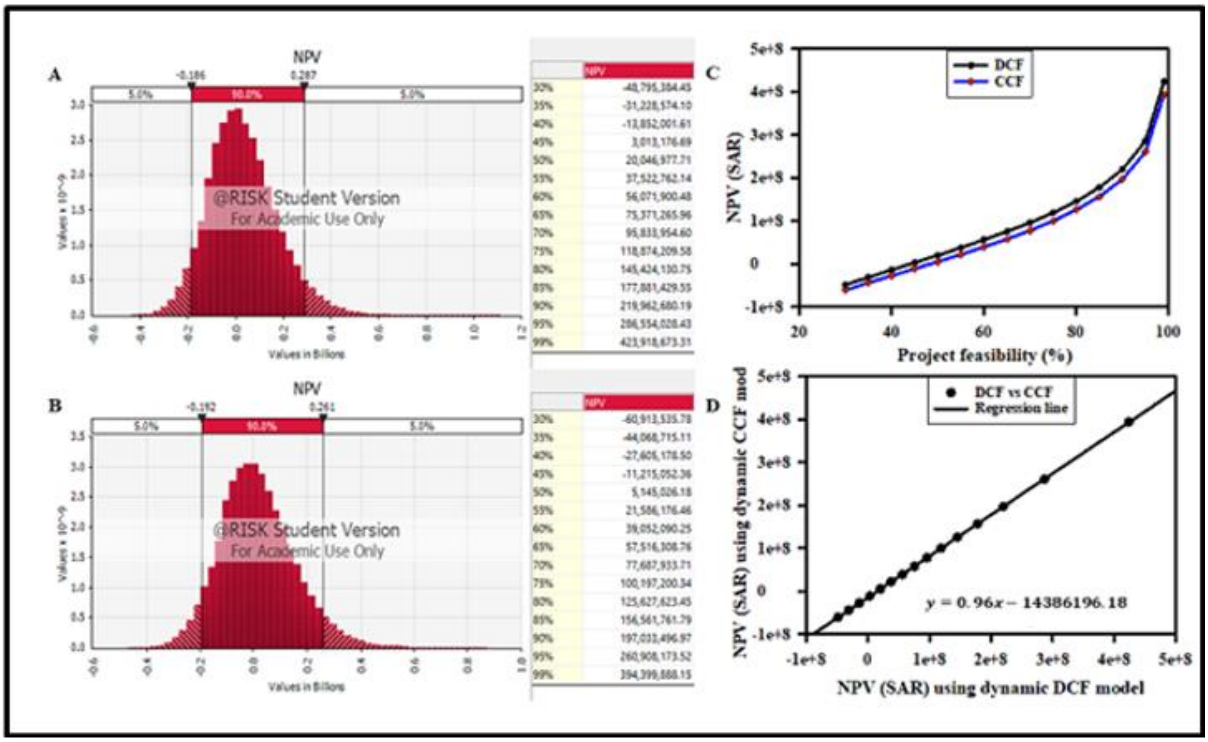

Figure 5. NPV (SAR) using dynamic model estimated after 10000 trials using @risk software (A) DCF, (B) CCF (C) DCF and CCF (D) Regression between DCF and CCF

Conclusions

This study introduces a novel continuous discount approach to evaluate the expected NPV of mine projects, using three scenarios of Continuous Cash Flow (CCF) models: increasing, decreasing, and uniform. The proposed CCF model was applied to a Saudi gold mine project, and its results were compared to a dynamic DCF model using Monte Carlo simulations of 10,000 iterations in @RISK software. Results show that the CCF model generally yields a lower NPV than the DCF model due to its exponential function accounting for all cash flows, irrespective of the project time frame. The CCF model demonstrated a strong linear relationship with the dynamic DCF model ($r^2 = 0.99$) and presents a useful alternative for NPV calculation in mining projects, warranting further validation in other feasibility-stage projects.

Author's Contribution:

Conceptualizations; Haitham Ahmed and Sefiu Adewuyi, Software; Sefiu Adewuyi, modelling, testing, and writing of first draft; Sefiu Adewuyi, writing review and editing; Sefiu Adewuyi, Hussin Ahmed, and Haitham Ahmed, supervision; Haitham Ahmed and Hussin Ahmed.

References

[1] Rudenno, V., The Mining Valuation Handbook. *John Wiley & Sons, Inc. Ltd.*, Australia (2012)

[2] Park, S.J. and Matunhire, I.I., Investigation of Factors Influencing the Determination of Discount Rate and the Application of Quantitative Methods For Discount Rate Using Risk Factors in the Minerals Industry. *South. African Inst. Min.* Metall. 6th South. African Base Met. Conf. 413–428 (2011).

[3] Saługa, P.W., Risk-adjusted discount rate and its components in evaluating hard coal projects at the feasibility stage. *Gospod. Surowcami Miner. / Miner. Resour. Manag*. 35, 63–74 (2019). https://doi.org/10.24425/gsm.2019.128530.

[4] Karaabat, V. M., Uğur, İ., and Erçelebi, S.G., Investment Risk Evaluation of Siirt Madenköy Copper Mine in Turkey. Bilge Int. J. Sci. Technol. Res. 3, 39–50 (2019). https://doi.org/10.30516/bilgesci.487621.

[5] Jaroni, M.S., Friedrich, B., and Letmathe, P., Economical Feasibility of Rare Earth Mining outside China. Minerals. 9, 1–18 (2019). https://doi.org/10.3390/min9100576.

[6] Tanchoco, J.M.A., Buck, J.R., and Leung, L.C., Modeling and discounting of continuous cash flows under risk. Eng. Costs Prod. Econ. 5, 205–216 (1981). https://doi.org/10.1016/0167-188X(81)90005-7.

[7] Brandimarte, P., *Handbook in Monte Carlo Simulation, Applications in Financial Engineering, Risk Management, and Economics. John Wiley & Sons, Inc (2014).*

[8] Zhang, X. and Wu, C., Continuous Cash Flow Payment, Theories and Practice Framework. *Emerg. Mark. Financ. Trade*. 00, 774–782 (2017). https://doi.org/10.1080/1540496X.2016.1241706

[9] Abdel Aziz, A.M. and Russell, A.D., Generalized Economic Modeling for Infrastructure and Capital Investment Projects. *J. Infrastruct. Syst.* 12, 18–32 (2006).

[10] Editor, C.M.J., Mining Valuation, Three steps beyond a static DCF model, http://www.canadianminingjournal.com/features/mining-valuation-three-steps-beyond-a-static-dcf-model/. Accessed 22 Februrary, 2020.

[11] Moyen, N., Slade, M., Uppal, R., Valuing risk and flexibility. *Resour. Policy*. 22, 63–74 (1996)

[12] Dehghani, H. and Ataee-pour, M., Determination of the effect of operating cost uncertainty on mining project evaluation. *Resour. Policy*. 37, 109–117 (2012). https://doi.org/10.1016/j.resourpol.2011.11.001

[13] Ugwuegbu, C.C., Segilola gold mine valuation using Monte Carlo simulation approach. *Miner. Econ*. 26, 39–46 (2013). https://doi.org/10.1007/s13563-013-0030-8

[14] Ardian, A. and Kumral, M., Dynamic discount rate through Ornstein-Uhlenbeck process for mining project valuation. In: IOP Conf. Series: Earth and Environmental Science. pp. 1–9. IOP Publishing (2018).

[15] Espinoza, D., Morris, J., Baroud, H., Bisogno, M., Cifuentes, A., Gentzoglanis, A., Luccioni, L., Rojo, J. and Vahedifard, F., The role of traditional discounted cash flows in the tragedy of the horizon: another inconvenient truth. *Mitig. Adapt. Strateg. Glob. Chang.* 1–18 (2019). https://doi.org/10.1007/s11027-019-09884-3.

[16] Pivorienė, A., Real Options and Discounted Cash Flow Analysis to Assess Strategic Investment Projects. *Econ. Bus.* 30, 91–101 (2017). https://doi.org/10.1515/eb-2017-0008.

[17] Zhang, X. and Wu, C., Continuous Cash Flow Payment: Theories and Practice Framework. Emerg. Mark. Financ. Trade. 54, 774–782 (2018). https://doi.org/https://doi.org/10.1080/1540496X.2016.1241706

[18] Pierce, P.E., Economic Analysis for U . S Geological Survey Coal Basin Assessments. , Reston, Virginia, U.S. (2019)

[19] London Metal Exchange, LME Gold, https://www.lme.com/Metals/Precious-metals/LME-Gold#tabIndex=2. Accessed 15 January, 2020.

[20] Hakmi, A., Saudi Arabia Mining Sector Market Brief. , Australia (2015).

[21] Said, R., and Daud, M.N., Utilising Monte Carlo Simulation for the Valuation of Mining Concessions. J. Des. Built Environ. 1, 1–17 (2000).

نمذجة التدفق النقدي المستمر للتقييم الاقتصادي وتحليل المخاطر في تخطيط المناجم باستخدام محاكاة مونت كارلو

هيثم أحمد و شفيع اولاكنلي ادويوحسين أ. م. أحمد

جامعة أريزونا ، توكسون ، أريزونا ، 85712 ، كلية الهندسة ، قسم التعدين والهندسة الجيولوجية

جامعة الملك عبد العزيز، جدة، كلية الهندسة، قسم هندسة التعدين

sadewuyi@arizona.edu

المستخلص: يُستخدم نموذج التدفق النقدي المخصوم (DCF) عادةً لتقدير صافي القيمة الحالية (NPV) للمشاريع المختلفة. ومع ذلك، في المشاريع التي تتسم بدرجات عالية من عدم اليقين وأطر زمنية طويلة، مثل مشاريع التعدين، قد يؤدي نموذج DCF إلى نتائج بعيدة عن الواقع. لمعالجة هذا التحدي، تقدم هذه الورقة نموذج التدفق النقدي المستمر (CCF) المصمم خصيصًا للتقييم الاقتصادي لمشاريع التعدين. تم تطبيق هذا النموذج لدراسة الجدوى الاقتصادية لمشروع تعدين الذهب في المملكة العربية السعودية. لأغراض التحليل المقارن، تم تقييم المشروع باستخدام كل من نموذج CCF المقترح ونموذج DCF التقليدي، حيث أُجريت ١٠٠٠٠ تكرار عبر برنامج Risk@ ومحاكاة باستخدام تقنية مونت كارلو. تشير النتائج إلى أن نموذج CCF ينتج قيمًا أقل لصافي القيمة الحالية مقارنةً بنموذج DCF، مع معامل ارتباط يبلغ ٠.٩٩ بين الطريقتين، مما يجعل نموذج CCF بديلاً فعّالاً لتحديد صافي القيمة الحالية المتوقعة في مشاريع التعدين ذات مستويات عالية من عدم اليقين.