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Pricing of Islamic Assets Based on Mean-Gini

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Abstract. With the emergence of ethical finance, Islamic finance has gained increasing significance as an investment option. This trend is not only observed in the predominantly Islamic countries but also in the finance industry in the West. This paper introduces the Mean Gini Sharī'ah Capital Asset Pricing Model (MG-SCAPM) for pricing Islamic assets. The proposed model incorporates key variables recommended in Islamic finance, such as zakāh, inflation rate, GDP rate, and Ṣukūk rate, as alternatives to the risk-free rate. Furthermore, a practical example is provided to demonstrate the implementation of MG-SCAPM and to aid Islamic investors in selecting the optimal model. In fact, based on our chosen data, the model that substitutes the risk-free rate with zakāh appears to be the most suitable for the current application.

Keywords: Islamic finance; Sharī'ah, Mean-Gini (MG), portfolio optimization, CAPM

JEL Classification : C53, C61, D53
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1. Introduction

The concept of portfolio optimization has consistently captivated the interest of investors and portfolio managers. Investors employ various approaches and strategies to simultaneously minimize risk and maximize returns in their portfolios. Markowitz (1952) stated that if a portfolio has the highest expected return for a specific variance, it is Mean Variance efficient. He was the first to investigate the foundations of modern portfolio theory. The Mean Variance (MV) Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964), has been widely used in finance. However, as highlighted by Bauder et al. (2021), the MV analysis is not without its limitations and shortcomings. One such drawback is the requirement of complete knowledge regarding the return distributions of all assets. In situations where certain asset distributions are unknown, the MV model can fail to accurately rank portfolios based on investor preferences. To address this issue, the Mean Gini (MG) portfolio rule emerges as an appealing alternative. It prevents investors from selecting portfolios that may be considered inadequate by offering a favorable environment for stochastic dominance analysis.

Yitzhaki (1982) underscored the advantages of Gini over Variance as a measure of variability. Shalit and Yitzhaki (1984) first introduced the MG approach in finance, unveiling the MG-CAPM for homogeneous investors. Thereafter, they proposed employing the MG technique instead of MV analysis for various reasons. Firstly, they observed that the efficient MG portfolio closely resembles the efficient MV portfolio when returns are assumed to follow a normal distribution, indicating that the MV model can be seen as a specific instance of the MG model. Secondly, in cases where the MV analysis fails to rank uncertain alternatives across a wide range of probability distributions, the MG analysis offers a reliable ranking of those alternatives, as indicated by Agouram and Lakhnati (2015). Based on this, we reach the conclusion that the MG-CAPM approach can effectively address portfolio risk management in an equilibrium market, as it remains unaffected by the distribution of returns, as stated in Agouram et al. (2020).

Islamic finance, which is rooted in Sharīʼah laws (see Nienhaus (2011)), represents a dynamically growing sector that encompasses a variety of financial products designed to adhere to the principles of Sharīʼah (refer to Cevik and Bugan (2018)). Thus, these products are specifically structured to comply with the ethical and moral standards outlined in Sharīʼah, contributing to the unique nature of Islamic finance. Ijasan et al. (2021) argued that Sharīʼah impedes the dynamism of assets, which may benefit the market during financial crises because of how strict these Sharīʼah laws are. The relationship between ethically based finance and financial performance is not easily identified. However, most research studies do not identify significant differences in risk-adjusted returns between moral and traditional investments Bauer et al. (2007).

Numerous studies have shown that Islamic finance demonstrates higher efficacy compared to conventional finance. Agouram et al. (2021); Arouri et al. (2016); Jawadi et al. (2014). This belief may be motivated by the fact that Islamic banking has strengthened in three aspects: liquidity, profitability, and efficiency (Salman and Nawaz (2018)). Indeed, Parashar (2010) documented that Islamic banking is more able to persist in an economic recession than conventional banks. According to Jawadi et al. (2014), the effect of the global economic crisis during the period 2008–2009 on Islamic markets is much less severe than on traditional markets, revealing that investors should keep a closer eye on Islamic financial products for essential alternative investments. Also, Djennas (2016) indicated that countries that apply Islamic finance principles are well-positioned to overcome different crisis scenarios and economic disasters. Adekoya et al. (2022) reported that Islamic markets are more resilient to COVID-19 than traditional markets (see Ben Hssain et al. (2022) for more about the effect of COVID-19).

From a theoretical perspective, Qoyum et al. (2022) provide confirmation that stakeholder theory, in particular, suggests that Islamic firms prioritize maximizing benefits for all stake- holders, rather than solely focusing on profit. Additionally, as stated by Hasan and Dridi (2011), Islamic banking demonstrated greater resilience compared to traditional banking during the subprime crisis (2007-2010). Further investigations (Guyot (2011)) have uncovered the substantial diversification advantages associated with investing in Islamic finance assets. Shahzad et al. (2019) put forward the idea that the Islamic bond index can serve as a valuable hedging tool within benchmark stock portfolios. Consequently, a number of prominent Western financial institutions have established Islamic subsidiaries and currently provide Islamic financial instruments to their clientele. One notable product is the Dow Jones Islamic Market Index - United States (DJIMI), which tracks the stocks of US companies whose operations and activities align with Islamic Sharīʼah principles.

Our research focuses on Islamic finance and is motivated not only by the fact that the literature on the pricing of Islamic assets is insufficient but also because Islamic finance has become a very important area for investment. Therefore, such an understanding enables portfolio managers and investors to make more sophisticated portfolio diversification and risk management strategies, considering their tendency to switch activities between conventional and ethical finance. As a result, originating an Islamic model for pricing assets and port-folio optimization can provide knowledge about the factors that drive ethical finance. This will ultimately optimize decision-making regarding asset pricing and permit better portfolio diversification during crises similar to the great recession (2007-2010) and COVID-19.

According to the findings of Boukhatem and Moussa (2018), governments should consider implementing proactive and beneficial economic and institutional policies aimed toward Islamic finance, especially since financial instability has emerged as a challenging issue for conventional economists and financiers to address (Zaman et al. (2019)).

Our findings make a valuable contribution to the current literature on Islamic finance by introducing an optimal CAPM framework based on Mean Gini. This framework allows investors to incorporate Sharīʼah concepts into their financial decision-making process, benefiting from the advantages offered by the Islamic system. Moreover, the implementation of MG-SCAPM can have a positive impact on the banking sector and the financial markets where the Islamic system is practiced. The application of this framework to the stock market raises questions about achieving investment objectives through Sharīʼah compliant stocks using the MG-SCAPM model we propose.

The paper is structured as follows: In the next section, we present literature reviews. Section three highlights the Mean-Gini and Portfolio analysis. Section four provides an overview of Mean-Gini analysis within the Islamic framework. Empirical results and discussions are presented in section five. Finally, the last section concludes.

2. Literature review

The Capital Asset Pricing Model has its origins in various advancements and contributions by several authors (Markowitz (1952); Sharpe (1964); Shalit and Yitzhaki (1984)), establishing itself as a predictive framework for estimating asset returns. However, despite the widespread acceptance of CAPM as a foundational model in equilibrium modeling, its practical application is hindered by various assumptions and limitations. Consequently, many researchers and authors have endeavored to enhance and adapt the CAPM to align with real-world conditions (Black (1986); Fama and French (1992, 2004)). One crucial aspect that is often overlooked in the CAPM is the presence of interest rates, which play a significant role in borrowing and lending funds, and are indicative of associated risks. In fact, during periods of instability, interest rates can contribute to the collapse or amplification of risk. Also, the CAPM only considers systematic risk, represented by the Classic Beta (see Bauder et al. (2021) for limitations of MV-CAPM). It overlooks other factors that can influence asset returns, such as company-specific factors, industry trends, or macroeconomic variables. These limitations can result in an oversimplified view of risk and return relationships. Some of these observations align with the principles of Islamic finance, which provide alternative approaches to address the issues in the financial system that arise from such phenomena.

In our current analysis, we have chosen not to delve into the distinctions between the interest rate and the risk-free rate. However, it is crucial to acknowledge that this topic holds significant importance and warrants thorough exploration in future papers. For the purposes of our study, we will adopt the prevailing convention that considers the interest rate to be equivalent to Riba. This association has already been extensively discussed and supported by numerous researchers in the field (we refer the reader to Al Rahahleh et al. (2019); Askari et al. (2013); Hakim and Hamid (2016); Hanif (2011); Shaikh (2010)).

The emergence of Islamic finance raises an important question regarding the compatibility of traditional investment methods and frameworks, such as Markowitz’s model strategy and the classic Capital Asset Pricing Model, with Sharīʾah principles. This question pertains to whether these conventional approaches can be employed to evaluate financial products in accordance with Sharīʾah law. As stated by Usmani (2007), Islamic financial institutions should be independent of this practice. According to Hakim and Hamid (2016), the CAPM can be applied in the context of Islamic finance. However, an inherent challenge arises due to the inclusion of risk-free interest as a variable in CAPM. Islam prohibits the payment or acceptance of interest charges, which poses a significant dilemma. Consequently, in Islamic financial systems, there is no direct equivalent to a risk-free interest rate from a theoretical standpoint. Capital holds significant importance in Islamic finance, but in line with Islamic principles, it cannot be disconnected from the obligation of zakāh. Capital encompasses both personal and corporate wealth. The fundamental objective of Islamic finance is to alleviate poverty, foster sustainable growth, and ensure full employment (Askari et al. (2013)). Differences in individual wealth can contribute to a societal divide between the affluent and the less privileged. In Islam, the obligation of zakāh requires Muslims to willingly share their wealth with those who are entitled to receive zakāh. Consequently, the principle of zakāh in Islam is intricately linked to the pursuit of social justice. It serves as a means to address the disparities in wealth distribution and promote a more equitable society. Therefore, zakāh holds great significance for every Muslim, including investors who profit from their activities. As a result, adjusting the risk-free return by zakāh in El-Ashker (1987) provides a theoretical basis for using the conventional CAPM in the pricing of Islamic assets. Hanif (2011) introduced an alternative approach to adjust the CAPM by replacing the risk-free rate with an inflation index. This concept recognizes that inflation reflects the risky conditions within a country, thus becoming a risk factor that investors should consider. Furthermore, this perspective aligns with the theory that posits a positive relationship between inflation and nominal interest rates, as highlighted by Asgharpur et al. (2007). Instead of eliminating the risk-free rate altogether, this modified model incorporates the inflation index as a substitute component. In simpler terms, when the expected return calculated by the CAPM exceeds the interest rate, it signifies that the investment return is capable of mitigating the impact of inflation. By integrating the inflation index, this adjusted CAPM accounts for the inflationary risk and its implications for investment performance. In this context, the preferred choice for an inflation index was the Consumer Price Index (CPI). Also, according to Shaikh (2010), an alternative approach for replacing the risk-free rate is to utilize the rate of Nominal Gross Domestic Product (NGDP), which serves as a representation of the average growth in the value of consumption within a society. The relationship is such that when consumption increases, so does GDP, and conversely, when consumption declines, GDP also decreases. In the context of finance, if a company generates sales during a specific financial reporting period, GDP would reflect the total sales generated by the country's population.

An alternative viewpoint regarding the substitution of the risk-free rate in Islamic finance arises from the understanding that the CAPM is constructed using US Treasuries as a proxy. Consequently, Sukuk issued by the Islamic Development Bank (IsDB) could be considered a new proxy for Sharī'ah compliant stocks, even in the absence of a risk-free rate in practical terms. Hakim and Hamid (2016) explores the use of Sukuk as a replacement for the risk-free rate, while Mustofa (2018) also highlights its significance in the context of Sharī'ah-compliant stocks. The subsequent question pertains to the distinction between the Sukuk rate and the interest rate. As explained in Hakim and Hamid (2016), the structure of Sukuk is specifically designed to adhere to religious principles and avoid the prohibition of Riba. Unlike traditional bonds, where interest is paid by the issuer to the bondholder as a form of loan, Sukuk represents investments in a range of different assets, particularly those that are free from any forbidden components of Ribat and non-halal business activities (Subekti and Rosadi (2020)). Our research contributes to the growing body of literature on the pricing of Islamic assets.

Our paper stands out as the first attempt to evaluate Islamic assets using the Mean Gini approach. By introducing this novel perspective, we aim to expand the understanding of Islamic finance and provide valuable insights into the pricing of these types of assets.

3. Mean-Gini and Portfolio analysis

3.1. Mean-Gini

To establish the problem's setup, let as consider an asset universe $\mathcal{I} = \{1, \dots, n\}$, where, $n \geqslant 2$ is an integer indicating the number of Islamic assets in the market, and we denote by Y_i the return of the i^{th} asset for a fixed period $(\Delta t \equiv [t-1, t])$ for each $i \in \mathcal{I}$. We define also a probability space$(\Omega, \mathcal{F}, \mathbb{P})$, which contains a random vector $Y = (Y_1, \dots, Y_n)$. In our context, the set of all portfolios can be as follows:

$$\mathcal{W} := \left\{ \omega = (\omega_i, \dots, \omega_n) \in \mathbb{R}^n \mid \sum_{i=1}^{n} \omega_i = 1 \right\} \tag{1}$$

where, ω_i denotes the weight of the i^{th} asset.

We will restrict the focus to portfolios in the subset $\mathcal{W}_+ = \mathcal{W} \cap \mathbb{R}_+^n$ when we are dealing with an Islamic portfolio, as short selling is not allowed in Islamic finance. Note that $\mathcal{Y}$ consists of all the linear combinations of the components of the vector $Y = (Y_1, \dots, Y_n)$ and their shifts by almost sure constants. In addition, throughout this paper, $X := X(\omega) \in \mathcal{Y}$ is a random variable modeling portfolio return and for a $p \in \mathbb{N}^*, L^p := \{X \mid \mathbb{E}[|X|^p] < \infty\}$. For every $X \in \mathcal{Y}, F_X$ denotes the Cumulative Distribution Function (cdf) of X.

Remark 1. We will take into consideration the fact that $\mathcal{W}$ is a compact convex set in $\mathbb{R}^n$. Since this property has a very significant effect on the existence and uniqueness of the solution to our main optimization problem.

Definition 3.1. (Shalit and Yitzhaki (1984)) The Gini can be defined as:

$$\text{Gini}(X) = \frac{1}{2}\int_a^b \int_a^b (u - x) f(u) f(x) du dx. \tag{2}$$

Where, f represent the density function of X, a and b represent, respectively, the lower and upper bounds of F_X, u and x are realization pairs of X.

Remark 2. In literature, there is many formulation of Gini. For example, we cite two of them:

$$\text{Gini}(X) = \int_a^b \big(1 - F_X(X)\big) dX - \int_a^b [1 - F_X(X)]^2 dX, \tag{3}$$

or, for a finite values of a,

$$\text{Gini}(X) = \mathbb{E}[X] - a - \int_a^b [1 - F_X(X)]^2 dX. \tag{4}$$

Proposition 3.2. We specify the following properties that make Gini an interesting risk measure (for the interpretation of those proprieties, we refer the reader to Frittelli and Gianin (2004)):

$\mathcal{P}\mathbf{1}$ Standardization: Gini $(b) = 0$ for all $b \in \mathbb{R}$.

$\mathcal{P}\mathbf{2}$ Location invariance: Gini $(X + b) =$ Gini (X) for all $X \in \mathcal{Y}$ and $b \in \mathbb{R}$.

$\mathcal{P}\mathbf{3}$ Positive homogeneity: Gin $i(\kappa X) = \kappa$Gini (X) for all $X \in \mathcal{Y}$ and $\kappa > 0$.

$\mathcal{P}\mathbf{4}$ Sub-additivity: Gini $(X + Z) \leq$ Gini $(X) +$ Gini (Z) for all $X, Z \in \mathcal{Y}$.

$\mathcal{P}\mathbf{5}$ Co-monotonic additivity: for every co-

monotonid[1] pair, $\in \mathcal{Y}$, Gini $(X+Z) =$ Gini (X) + Gini (Z).
Additionally, under sub-additivity and positive homogeneity is equivalent to the following property.
$\boldsymbol{\mathcal{P}6}$ Convexity: it hold Gini $(\lambda X + (1-\lambda)Z) \leq \lambda$Gini $(X) + (1-\lambda)$Gini (Z) for every $\lambda \in [0,1]$ and $X, Z \in \mathcal{Y}$.
Definition 3.3. A functional is said to be a measure of variability if it satisfies properties $\boldsymbol{\mathcal{P}1}$ and $\boldsymbol{\mathcal{P}2}$ and it is a coherent measure of variability if it further satisfies $\boldsymbol{\mathcal{P}3}$ and $\boldsymbol{\mathcal{P}4}$ (we refer to Berkhouch et al. (2018) for an overview on measure of variability).
Definition 3.4. Let $X, Z \in L^1$, we say that Z is Second-order Stochastically Dominated by X, succinctly $Z \preceq_{SSD} X$, if $\mathbb{E}[G(Z)] \leq \mathbb{E}[G(X)]$ for every increasing convex functions G. If in addition, $\mathbb{E}[X] = \mathbb{E}[Z]$, then we say that Z is smaller than X in convex order, succinctly $Z \preceq_{CX} X$.

Remark 3. The most classical measures used to measure variability are the variance and the standard deviation. Nevertheless, these are not co-monotonically additive. However, Gini is a coherent measure of variability Furman et al. (2017). Furthermore, it is CX-monotone while $\preceq_{CX}$ implies $\preceq_{SSD}$, therefore Gini is SSD-monotone.

3.2. Gini for Portfolio analysis

In the Mean-Gini model, the performance and risk of a particular portfolio are influenced by the individual asset weights within the portfolio and the correlation among these assets. Presented below are several formulas related to the *Gini* index for portfolio analysis:

[1] X and Z are said to be co-monotonic if: $\forall(\omega,\omega') \in \Omega^2, (X(\omega') - Z(\omega))(X(\omega) - Z(\omega')) \geqslant 0$.

$$\mathrm{Gini}(X) = 2\int_a^b X\left[F_X(X) - \frac{1}{2}\right] f(X)dX = 2\,\mathrm{Cov}[X, F_X(X)], \tag{5}$$

where X, from now on, is the return of an Islamic portfolio, defined as follows:

$$X = \sum_{i=1}^{n} \omega_i(1 - z_i)(1 - \nu_i)Y_i, \tag{6}$$

$$\text{for} \sum_{i=1}^{n} \omega_i = 1. \tag{7}$$

Where,

- $\omega_i \geq 0$ for all $i = 1, \ldots, n$, to avoid short sales which are not allowable in Islamic finance.
- z_i denotes zakāh rate (for our case here we will take $z_i = 0$ if $Y_i \leq 0$ and $z_i > 0$ if $Y_i > 0$)
- ν_i denotes individual purification.

Taking into account these new variables, the Gini representation for an Islamic portfolio $(\omega \in \mathcal{W}_+)$, can be formulated as the following:

$$\mathrm{Gini}(X) = 2\sum_{i=1}^{n} \omega_i(1 - z_i)(1 - \nu_i)\,\mathrm{Cov}[Y_i, F_X(X)] \tag{8}$$

Remark 4. We notice from equation (8) that for Islamic case the portfolio risk cannot be decomposed into just a simple weighted sum of the covariance between the cumulative distribution function and the variablesY_i, but there are also new parameters (z_i and ν_i) that can change the classical proceedings about how we manage the portfolio risks.

The variance of an Islamic portfolio can be written as in equation (9). We noticed the emergence of the parameters (z_i and ν_i), which we have already mentioned for the Gini case.

$$\mathrm{VAR}(X) = \sum_{i=1}^{n} \omega_i(1 - z_i)(1 - \nu_i)\,\mathrm{Cov}[Y_i, X] \tag{9}$$

For the Islamic case, we revealed that the main distinction between the two models (Gini and variance) is the fact that in equation (8) the risk is expressed by F_X, while in (9) it is given by its return (X). Moreover, there are some more parameters for both models. We also point out from Furman et al. (2017) that variance is not a coherent measure of variability (Definition 2.3). This might lead us to investigate the mean-Gini model more closely.

4. Mean-Gini under Islamic framework

In this section, we formulate the asset-pricing model of an Islamic investor using the Mean Gini efficient portfolio. We recall that conventional CAPM relation between expected return and risk is given as follows:

$$\mathbb{E}[Y_i] = r_f + \left(\mathbb{E}[X_m] - r_f\right)\frac{\mathrm{Cov}\,[Y_i, X_m]}{\sigma_m^2} \tag{10}$$

where,

- $\mathbb{E}[Y_i]$ is the expected return of the i^{th} asset
- r_f is the risk-free rate
- σ_m^2 represents the market's return variance
- $\mathbb{E}[X_m]$ is the expected return of the market.

The formula depicted as (10) serves as a valuable reference for determining returns within a conventional setting. However, when examined within the framework of Islamic Sharī’ah law, where the notion of fixed or risk-free returns does not exist, it raises doubts. According to Sharī’ah principles, the concept of interest is strictly prohibited and deemed as Ribat, which is not permissible.

From one point of view, if we eliminate the interest factor, we get an adapted CAPM, called the Sharī’ah capital asset pricing model (SCAPM):

$$\mathbb{E}[Y_i] = \mathbb{E}[X_m]\frac{\mathrm{Cov}\,[Y_i, X_m]}{\sigma_m^2}. \tag{11}$$

Looking at it from a different perspective, several Islamic scholars and researchers have proposed substituting r_f with alternative factors. For instance, it has been suggested in El-Ashker (1987) that zakāh could serve as a replacement, while others have proposed considering factors such as the inflation rate (Hanif (2011)) or the nominal growth of GDP (Shaikh (2010)). Consequently, the SCAPM can be expressed as follows:

$$\mathbb{E}[Y_i] = k_{\text{islamic}} + [\mathbb{E}[X_m] - k_{\text{islamic}}]\frac{\mathrm{Cov}\,[Y_i, X_m]}{\sigma_m^2}, \tag{12}$$

where,

- k_{islamic} is the alternative of r_f in Islamic finance.

The fundamental assumptions of the classical MG-CAPM, as outlined in Shalit and Yitzhaki (1984), form the basis for the forthcoming analysis. However, in this analysis, the classical risk-free rate will be replaced. Each Islamic investor will determine their optimal portfolio by selecting non-negative weights that minimize the Gini index of a portfolio at a given expected return. The return of an Islamic portfolio for an investor j will be given, from now on, as follows:

$$X_j = \sum_{i=1}^{n} \omega_i^j (1 - z_i)(1 - v_i)Y_i, \tag{13}$$

$$\text{where, } \sum_{i=1}^{n} \omega_i^j = 1,\ \omega_i^j \geq 0, i = 1, \dots, n. \tag{14}$$

The primary objective for an Islamic investor is to minimize the Gini index (referred to as Islamic Gini or *IGini* hereafter) of their portfolio at a specific expected return $\mathbb{E}[X_j]$. Consequently, portfolio optimization, guided by *IGini*, can be achieved by solving the following problem:

$$\begin{aligned}
&\underset{\omega}{\text{Minimize}}\ \text{IGini}\left(X_j\right)\\
&\text{s.t. } \mathbb{E}\left[X_j\right] = \sum_{i=1}^{n} \omega_i^j (1-z_i)(1-\nu_i)\mathbb{E}[Y_i],\\
&\sum_{i=1}^{n} \omega_i^j = 1,\\
&\omega_i^j \geqslant 0,\ i \in \{1, \dots, n\}.
\end{aligned}$$

Recall that IGini of an Islamic portfolio can be represented as follows:

$$\text{IGini}\left(X_j\right) = 2\sum_{i=1}^{n} \omega_i^j (1-z_i)(1-\nu_i)\,\text{Cov}\left[Y_i, F_{X_j}\left(X_j\right)\right]. \tag{15}$$

Then, the main problem can be formulated as the following:

$$\begin{aligned}
&\underset{\substack{\omega\\ \omega}}{\text{Minimize}}\ 2\sum_{i=1}^{n} \omega_i^j (1-z_i)(1-\nu_i)\text{Cov}\left[Y_i, F_{X_j}\left(X_j\right)\right]\\
&\text{s.t. } \mathbb{E}\left[X_j\right] = \sum_{i=1}^{n} \omega_i^j (1-z_i)(1-\nu_i)\mathbb{E}[Y_i]\\
&\sum_{i=1}^{n} \omega_i^j = 1\\
&\omega_i^j \geqslant 0,\ i \in \{1, \dots, n\}.
\end{aligned}$$

Let, λ_j be the Lagrange multiplier associated with an Islamic investor j. Thus, for all $\in \{1, \dots, n\}$, the necessary conditions for a minimum are simply:

$$2(1-z_i)(1-\nu_i)\,\text{Cov}\left[Y_i, F_{X_j}\left(X_j\right)\right] + 2\sum_{k=1}^{n} \omega_k^j (1-z_k)(1-\nu_k)\frac{\partial \text{Cov}\left[Y_k, F_{X_j}\left(X_j\right)\right]}{\partial \omega_i^j} = \lambda_j (1-z_i)(1-\nu_i)\mathbb{E}[Y_i], \tag{16}$$

According to Shalit and Yitzhaki (1984), the quasi-convexity of the Gini function ensures second-order conditions, and by propriety $\boldsymbol{\mathcal{P}3}$ of the Gini coefficient and equation (15), we know that IGini is homogeneous[2] of degree one in ω_i^j. Therefore by Euler theorem: of X.

[2] Let $H\left(X_j, F_{X_j}\left(X_j\right)\right) = \text{Gini}\left(X_j\right)$ and $k \in \mathbb{R}$, we know that multiplying X_j and $F_{X_j}\left(X_j\right)$ by k will not change cdf

$$\begin{aligned}
\text{Gini}\left(kX_j\right) = H\left(kX_j, kF_{X_j}\left(X_j\right)\right) &= 2\text{Cov}\left[kX_j, kF_{X_j}\left(X_j\right)\right]\\
&= 2\text{Cov}\left[kX_j, F_{X_j}\left(X_j\right)\right]\\
&= 2k\text{Cov}\left[X_j, F_{X_j}\left(X_j\right)\right]\\
&= k^1 H\left(X_j, F_{X_j}\left(X_j\right)\right)\\
\text{Gini}\left(kX_j\right) &= k^1 \text{Gini}\left(X_j\right)
\end{aligned}$$

$$\mathrm{IGini}(X_j) = \sum_{i=1}^{n} \omega_i^j \frac{\partial\, \mathrm{IGini}(X_j)}{\partial \omega_i^j} \qquad (17)$$

Where,

$$\omega_i^j \frac{\partial \mathrm{IGini}\,(X_j)}{\partial \omega_i^j} = 2\omega_i^j (1 - z_i)(1 - \nu_i)\mathrm{Cov}\left[Y_i, F_{X_j}(X_j)\right] + 2\sum_{k=1}^{n} \omega_k^j \omega_i^j (1 - z_k)(1 - \nu_k) \frac{\partial \mathrm{Cov}\left[Y_k, F_{X_j}(X_j)\right]}{\partial \omega_i^j}$$

or

$$\mathrm{IGini}\,(X_j) = \sum_{i=1}^{n} \omega_i^j \frac{\partial \mathrm{IGini}\,(X_j)}{\partial \omega_i^j} = 2\sum_{i=1}^{n} \omega_i^j (1 - z_i)(1 - \nu_i)\mathrm{Cov}\left[Y_i, F_{X_j}(X_j)\right] + 2\sum_{i=1}^{n}\sum_{k=1}^{n} \omega_i^j \omega_k^j (1 - z_k)(1 - \nu_k) \frac{\partial \mathrm{Cov}\left[Y_k, F_{X_j}(X_j)\right]}{\partial \omega_i^j} \qquad (18)$$

Taking again the equation (15), we can eliminate the double sum of the right-hand side by multiplying the two sides by the part ω_i^j in (16) and we will obtain

$$2\omega_i^j (1 - z_i)(1 - \nu_i)\mathrm{Cov}\left[Y_i, F_{X_j}(X_j)\right] + 2\sum_{k=1}^{n} \omega_k^j \omega_i^j (1 - \nu_k)(1 - z_k) \frac{\partial \mathrm{Cov}\left[Y_k, F_{X_j}(X_j)\right]}{\partial \omega_i^j} = \lambda_j \omega_i^j (1 - z_i)(1 - \nu_i)\mathbb{E}[Y_i].$$

If we add up all the assets n, then for each Islamic investor we have:

$$\mathrm{IGini}\,(X_j) = \lambda_j \sum_{i=1}^{n} \omega_i^j (1 - z_i)(1 - \nu_i)\mathbb{E}[Y_i] = \lambda_j \mathbb{E}[X_j]$$

and we have obtained the relation between risk (expressed by Gini) of the portfolio and its expected return as:

$$\mathrm{IGini}(X_j) = \lambda_j \mathbb{E}[X_j] \qquad (19)$$

Let’s consider a scenario where there exists a market comprising of homogeneous Islamic investors who have identical investment options, share a risk-averse attitude, and aim to minimize the Gini coefficient of their portfolios while achieving their desired expected returns. In such a situation, Equation (19) will hold true and be applicable to all Islamic investors within that market.

$$\frac{1}{\lambda} = \frac{\mathbb{E}[X_m]}{\mathrm{IGini}\,(X_m)}. \qquad (20)$$

Where, IGini (X_m) is the Gini coefficient and $\mathbb{E}[X_m]$ is the expected return, on the market portfolio. Also in the Islamic case, because the optimal position, $F_j(X_j)$ of investor j, stays the same, the Gini coefficient of the portfolio that is held by investor j is equal to IGini (X_m). Thus, $F_j(X_j) = F_m(X_m)$ for all Islamic investors j (for further details see Shalit and Yitzhaki (1984). Thus from (16), we obtain:

$$2(1-z_i)(1-\nu_i)\,\mathrm{Cov}\big[Y_i, F_{X_m}(X_m)\big] = \lambda_j \mathbb{E}[Y_i](1-z_i)(1-\nu_i), \qquad (21)$$

while in the equilibrium,

$$2\sum_{k=1}^{n}(1-z_k)(1-\nu_k)\omega_k^j \frac{\partial \mathrm{Cov}\,[Y_k, F_m(X_m)]}{\partial \omega_j} = 0.$$

Therefore, the equilibrium for every asset $i = 1, \dots, n$ and Islamic investor j becomes as the following:

$$\mathbb{E}[Y_i] = \frac{2\mathrm{Cov}\,\big[Y_i, F_{X_m}(X_m)\big]}{\mathrm{Gini}\,(X_m)}\mathbb{E}[X_m]. \qquad (22)$$

Then, equation (22) represents the SCAPM valuation relationship for a market of an Islamic investor using the mean-Gini approach. Furthermore, to highlight the dependence between systematic risk and expected return, in the Islamic case, considering that

$$2\mathrm{Cov}\,[Y_i, F_m(X_m)] = \alpha_{im}\mathrm{Gini}\,(Y_i),$$

where,

$$\alpha_{im} = \frac{\mathrm{Cov}\,[Y_i, F_m(X_m)]}{\mathrm{Cov}\,\big[Y_i, F_i\big((1-z_i)(1-\nu_i)Y_i\big)\big]},$$

we recall that

$$\mathrm{Gini}(Y_i) = 2(1-z_i)(1-\nu_i)\,\mathrm{Cov}\big[Y_i, F_i\big((1-z_i)(1-\nu_i)Y_i\big)\big] \qquad (23)$$

thus,

$$\mathbb{E}[Y_i] = \mathbb{E}[X_m]\alpha_{im}\frac{\mathrm{Gini}(Y_i)}{(1-\nu_i)(1-z_i)\,\mathrm{Gini}(X_m)} \qquad (24)$$

Therefore, the Islamic asset 's beta can be represented as follows:

$$\beta_{Gini,i} = \alpha_{im}\frac{\mathrm{Gini}\,(Y_i)}{(1-\nu_i)(1-z_i)\mathrm{Gini}\,(X_m)} = \frac{2\mathrm{Cov}\,\big[Y_i, F_{X_m}(X_m)\big]}{\mathrm{Gini}\,(X_m)}. \qquad (25)$$

We notice that, $\beta_{\mathrm{Gini},i}$, in this context, refers to the level of the responsiveness of the return of i^{th} asset to the movement in the market (Islamic market), and α_{im} is the ratio of total risk that cannot be decreased by the market without a decline in the expected return.

4.1. Mean-Gini Sharī'ah capital asset pricing models: MG-SCAPM

It is very important to note that the originally suggested equation for the MG-SCAPM is as follows.

$$\mathbb{E}[Y_i] = \beta_{Gini,i}\mathbb{E}[X_m]. \quad (26)$$

Nevertheless, an alternative approach exists for adjusting the MG-CAPM. It involves substituting the risk-free rate with its equivalent in Islamic finance, as proposed by Islamic researchers. Consequently, the MG-SCAPM formula, following the suggested adaptation in El-Ashker (1987), can be expressed as follows:

$$\mathbb{E}[Y_i] = z_i + (\mathbb{E}[X_m] - z_i)\beta_{\text{Gini},i}. \quad (27)$$

The inclusion of the inflation factor is an integral part of the model, as highlighted by Hanif (2011). Thus, the equivalence of MG-SCAM can be represented as follows:

$$\mathbb{E}[Y_i] = r_{\text{inflation}} + (\mathbb{E}[X_m] - r_{\text{inflation}})\beta_{\text{Gini},i}. \quad (28)$$

In addition, the MG-SCAPM takes the following form, based on the adaption suggested in Shaikh (2010).

$$\mathbb{E}[Y_i] = r_{NGDP} + (\mathbb{E}[X_m] - r_{NGDP})\beta_{Gini,i}. \quad (29)$$

We can generate an additional version of the MG-CAPM by taking the risk-free rate and replacing it with the inflation rate Hakim et al. (2016).

$$\mathbb{E}[Y_i] = r_{\text{sukuk}} + (\mathbb{E}[X_m] - r_{\text{sukuk}})\beta_{\text{Gini},i}. \quad (30)$$

The model recently developed by Derbali et al. (2017) can be adapted to the Mean-Gini case as follows:

$$\mathbb{E}[Y_i] = \frac{R_s}{1-\nu_i} + \left(\mathbb{E}[X_m] - \frac{R_s}{1-\nu_m}\right)\beta_{\text{Gini}\ i,i}, \quad (31)$$

where,

- R_s is the Sukuk profit rate
- ν_m is either the market purification rate or an ideal purification factor.

4.2. MG-SCAPM regression models

Finally, within the context of regression analysis, the MG-SCAPM can be implemented in five different versions as outlined below.

- Model I :

$$\mathbb{E}[Y_i] = \alpha_{i,1} + \mathbb{E}[X_m]\beta_{\text{Gini},i} + \epsilon_{i,1} \quad (32)$$

- Model II :

$$\mathbb{E}[Y_i] - z_i = \alpha_{i,2} + (\mathbb{E}[X_m] - z_i)\beta_{\text{Gini},i} + \epsilon_{i,2} \quad (33)$$

- Model III :

$$\mathbb{E}[Y_i] - r_{\text{inflation}} = \alpha_{i,3} + (\mathbb{E}[X_m] - r_{\text{inflation}})\beta_{\text{Gini},i} + \epsilon_{i,3} \quad (34)$$

- Model IV :

$$\mathbb{E}[Y_i] - r_{NGDP} = \alpha_{i,4} + (\mathbb{E}[X_m] - r_{NGDP})\beta_{Gini,i} + \epsilon_{i,4} \quad (35)$$

- Model V :

$$\mathbb{E}[Y_i] - r_{\text{sukuk}} = \alpha_{i,5} + (\mathbb{E}[X_m] - r_{\text{sukuk}})\beta_{\text{Gini},i} + \epsilon_{i,5} \quad (36)$$

- Model VI :

$$\mathbb{E}[Y_i] - \frac{R_s}{1-\nu_i} = \alpha_{i,6} + \left(\mathbb{E}[X_m] - \frac{R_s}{1-\nu_m}\right)\beta_{Gini,i} + \epsilon_{i,6}. \qquad (37)$$

Where, ϵ_i is the corresponding error terms and α_i represents the time series regression's intercept term.

5. Empirical results and discussions

This section is expected to serve as a proof of concept to show the importance of Islamic investment and for the implementation of the MG-SCAPM. And to answer the question, why MG-SCAPM rather than MV-SCAPM? We investigate empirically the daily rates of return from the top 10 Islamic constituents in Bursa Malaysia (Table 1). We experimentally compare Markowitz's Mean–Variance model and the Islamic CAPM based on the Mean-Gini. The data collected is the closing price from July 2012 to December 2021. As a stand-in for market portfolio returns, the Kuala Lumpur Composite Index (KLCI) returns are employed. In our case, to apply the SCAPM, we need to consider information such as the inflation rate, zakāh, purification, GDP rate, and Sukuk rate.

Table 1: Top 10 Islamic constituents in Bursa Malaysia

Assets	Constituent	Sector
TNB	Tenaga Nasional	Electricity
PCG	PETRONAS Chemicals Group Bhd	Chemicals
PMB	Press Metal Aluminium Holdings	Industrial Metals and Mining
IHH	IHH Healthcare	Health Care Providers
AXIA	Axiata Group Bhd	Telecommunications Service Providers
DIGI	Digi.com	Telecommunications Service Providers
TGP	Top Glove Corp	Medical Equipment and Services
SDP	Sime Darby Plantation	Food Producers
MAX	Maxis Bhd	Telecommunications Service Providers
DIG	Dialog Group	Oil Gas and Coal

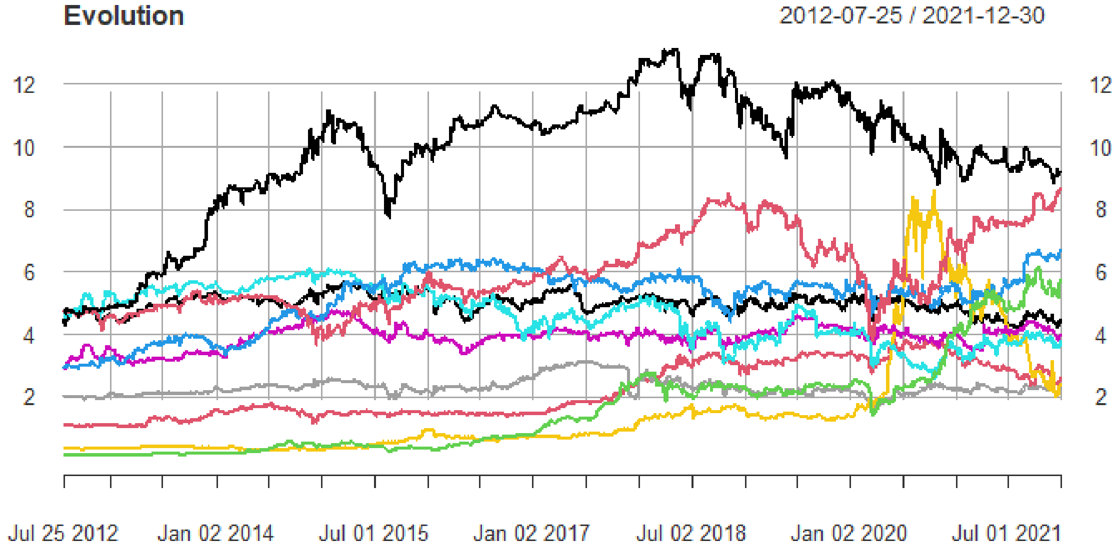
Source: Authors' Own

The returns of share *i* at time t are calculated using the following formula:

$$\mathrm{X} = \log\left(\frac{p_t}{p_{t-1}}\right) \times 100 \qquad (38)$$

where, P_t and P_{t-1} are, respectively, the prices of the chosen Islamic share on day t and $t-1$.

Figure 1. Evolution of close prices of the chosen constituents over the period: July 2012-Dec 2021.



Source: Authors' Own

Firstly, let's consider the great recession of 2008. Research conducted by Jawadi et al. (2014), Hasan and Dridi (2011) reveals that Islamic finance exhibited strong performance in comparison to conventional finance during this global financial crisis. Moreover, our findings (see also Dharani et al. (2022)), confirm that the studied shares were not significantly impacted by COVID-19 in March 2020. This observation suggests that Islamic instruments potentially offer greater resilience and security during times of pandemic crises. As depicted in Figure 5, the value of the chosen Islamic companies has demonstrated a positive upward trend over the analyzed period. This observation aligns with the findings of Abedifar et al. (2016), which suggest that Islamic finance is experiencing an annual growth rate of around 15-20%. Concerning research, it was noted by Ghlamallah et al. (2021) that around 1500 research publications on Islamic finance were released between 1979 and 2018. Consequently, we assert that a deeper comprehension of risk factors can potentially accelerate the advancement of ethical investing.

5.1. Descriptive statistics

Table 2: Descriptive statistics of return

	TNB	PCG	PMB	IHH	AXIA	DIGI	TGP	SDP	MAX	DIG
Mean	**0.0289**	**0.0259**	**0.1613**	**0.0362**	**-0.0080**	**0.0139**	**0.0870**	**0.0054**	**0.0000**	**0.0390**
SD	0.0112	0.0156	0.0220	0.0126	0.0168	0.0127	0.0247	0.0159	0.0122	0.0169
Skew	11.05	33.8527	9.1356	7.2103	12.7753	17.5376	15.14	107.24	8.48	9.87
Kurt	0.5777	-1.7657	-0.0089	0.2979	-0.4650	1.1838	0.1669	-4.5068	-0.1125	-0.3726
Max	0.0877	0.0893	0.1313	0.0844	0.1385	0.1734	0.1892	0.1062	0.1054	0.0984
Med	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Min	-0.0945	-0.2501	-0.2214	-0.0733	-0.1717	-0.0611	-0.2816	-0.3516	-0.0938	-0.1677
J-B	11826	110912	7986.9	5008.4	15704	29979	21961	1109083	6886.9	9383.7

Prob	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Obs	2308	2308	2308	2308	2308	2308	2308	2308	2308	2308

Source: Authors' Own

Upon examining Table 2, it can be observed that the kurtosis (kurt) for each share is below 3, the skewness (skew) is non-zero, and the mean and median (med) values differ. These findings collectively suggest that the studied series do not conform to a normal distribution. The Jarque-Bera (J-B) statistics provide further evidence against the assumption of a normal distribution. The findings reveal that Press Metal Aluminium Holdings (PMB) yields a higher return of 0.1613, albeit accompanied by the highest standard deviation of 2.2%. In contrast, Axiata Group Bhd exhibits a lower return of -0.008 along with a relatively significant standard deviation of 1.68%. On the other hand, Tenaga Nasional (TNB) demonstrates the lowest standard deviation of 1.12% coupled with a relatively higher return of 0.0289. Considering these results, we propose that the studied Islamic institutions have emerged as crucial investment opportunities due to their positive average returns over the past 10 years (2012–2021). The only exception to this trend is Axiata Group Bhd.

Table 3: Correlation between shares

	TNB	PCG	PMB	IHH	AXIA	DIGI	TGP	SDP	MAX	DIG
TNB	1.0000	0.2324	0.2252	0.2138	0.2286	0.2308	**0.0297**	0.2049	0.2197	0.2305
PCG	0.2324	1.0000	0.2394	0.1752	0.2126	0.2164	0.0355	0.1975	0.1811	0.2799
PMB	0.2252	0.2394	1.0000	0.1382	0.1619	0.1561	0.0708	0.1560	0.1756	0.2356
IHH	0.2138	0.1752	0.1382	1.0000	0.2036	0.1911	0.0725	0.1472	0.1943	0.1424
AXIA	0.2286	0.2126	0.1619	0.2036	1.0000	**0.3940**	0.0367	0.1839	0.2940	0.1939
DIGI	0.2308	0.2164	0.1561	0.1911	**0.3940**	1.0000	0.0544	0.1609	0.3432	0.1651
DIGI	**0.0297**	0.0355	0.0708	0.0725	0.0367	0.0544	1.0000	0.0543	0.0631	0.0911
SDP	0.2049	0.1975	0.1560	0.1472	0.1839	0.1609	0.0543	1.0000	0.1221	0.2107
MAX	0.2197	0.1811	0.1756	0.1943	0.2940	0.3432	0.0631	0.1221	1.0000	0.1364
DIG	0.2305	0.2799	0.2356	0.1424	0.1939	0.1651	0.0911	0.2107	0.1364	1.0000

Source: Authors' Own

Based on Table 3, we can observe that there the highest correlation coefficient is 39.4% between Axiata Group Bhd and Digi.com. On the other hand, there is a low correlation coefficient of 2.97% between Top Glove Corp and Tenaga Nasional, suggesting a weaker relationship between them. This implies that diversification strategies still play a significant role in reducing risk, even during crisis periods, for the chosen companies.

5.2 .Sharīʼah CAPM based on classic Beta and Gini Beta

We will now compare different Sharīʼah CAPMs in this subsection, some based on variance (classical Beta), and some based on the Gini coefficient (Gini Beta). Our goal is to determine the optimal model that can provide the most precise estimates of expected return and risk (standard deviation).

Table 4: Gini Beta (Beta1) and classic Beta (Beta2)

	TNB	PCG	PMB	IHH	AXIA	DIGI	TGP	SDP	MAX	DIG
Beta1	0,8379	1,1320	1,1814	0,7524	1,2895	0,9367	0,9167	0,9235	0,8209	1,0133
Beta2	0,8533	1,2304	1,2745	0,7251	1,2269	0,8513	0,8718	0,8913	0,7709	1,0430

Source: Authors' Own

Table 4 presents the results of our initial task, which involved computing the Gini Beta and classical Beta. In the subsequent Tables 5, 6, 7, and 8, we will proceed to estimate the expected return (Mean) and standard deviation (SD) for comparison with the actual results provided in Table 2. This analysis aims to assess the accuracy of our estimations in predicting returns and risk.

The estimated expected returns and standard deviation are presented in Table 5. In order to determine the optimal model, we assessed the errors associated with the expected returns of each stock for both the Mean Gini and Mean Variance models. It is observed that 50% of the errors generated by the classical model are smaller than those generated by the Gini model. As a result, we remain neutral regarding the performance of the models in this particular case, where $k_{islamic} \coloneqq 0$.

Table 5: Sharīʼah CAPM for $\mathbf{k_{islamic} \coloneqq 0}$

	Model I: $\mathbb{E}[Y_i] = \alpha_{i,1} + \mathbb{E}[X_m]\beta_{\text{Gini},i} + \epsilon_{i,1}$						
	$\beta_{\text{Gini},i} = \beta_i$			$\beta_{\text{classic},i} = \beta_i$			
Assets	Mean(%)	SD	Error 1[3]	Mean(%)	SD	Error 2[4]	Comparison
TNB	0,0243	0,0093	0,0047	0,0247	0,0095	0,0042	Error1 >Error2
PCG	0,0294	0,0177	0,0034	0,0221	0,0133	0,0038	Error1 <Error2
PMB	0,1905	0,0259	0,0293	0,1376	0,0187	0,0237	Error1> Error2
IHH	0,0272	0,0095	0,0090	0,0309	0,0108	0,0053	Error1 >Error2
AXIA	-0,0103	0,0217	0,0023	-0,0068	0,0144	0,0012	Error1> Error2
DIGI	0,0130	0,0119	0,0009	0,0118	0,0108	0,0020	Error1 <Error2
TGP	0,0798	0,0227	0,0072	0,0743	0,0211	0,0128	Error1 <Error2
SDP	0,0050	0,0147	0,0004	0,0046	0,0136	0,0008	Error1 <Error2
MAX	0,0000	0,0100	0,0000	0,0000	0,0104	0,0000	Error1 >Error2
DIG	0,0395	0,0171	0,0005	0,0333	0,0144	0,0057	Error1< Error2

Source: Authors' Own

3 Error 1 = real expected return - estimated expected return (for $\beta_{\text{Gini},i} = \beta_i$ and $k_{islamic} \coloneqq 0$).
4 Error 2 = real expected return - estimated expected return (for $\beta_{\text{classic},i} = \beta_i$ and $k_{islamic} \coloneqq 0$).

When examining the second case depicted in Table 6, an intriguing observation arises. Approximately 90% of the errors generated by the Gini model are smaller in magnitude than those generated by the classical model. This compelling evidence suggests that the Gini model provides a greater level of information and insight compared to the classical model, particularly in scenarios where $k_{islamic}$= zakāh. Thus, we can confidently assert that the Gini model offers enhanced accuracy and a more comprehensive understanding of the dynamics at play when incorporating the concept of zakāh as a substitution for the risk-free rate.

Table 6: Sharī'ah CAPM for $\boldsymbol{k_{islamic}} := \boldsymbol{zak\bar{a}h}$

	Model II: $\mathbb{E}[Y_i] - z_i = \alpha_{i,2} + (\mathbb{E}[X_m] - z_i)\beta_{\text{Gini},i} + \epsilon_{i,2}$						
	$\beta_{\text{Gini},i} = \beta_i$			$\beta_{\text{classic},i} = \beta_i$			
Assets	Mean (%)	SD	Error 3[5]	Mean (%)	SD	Error 4[6]	Comparison
TNB	0,0283	0,0093	0,0006	0,0284	0,0095	0,0006	Error 3= Error 4
PCG	0,0261	0,0177	0,0001	0,0262	0,0192	0,0002	Error 3< Error 4
PMB	0,1860	0,0259	0,0247	0,1987	0,0280	0,0374	Error 3< Error 4
IHH	0,0334	0,0095	0,0028	0,0331	0,0091	0,0031	Error 3<Error 4
AXIA	-0,0175	0,0217	0,0095	-0,0154	0,0206	0,0075	Error 3<Error 4
DIGI	0,0146	0,0119	0,0007	0,0155	0,0108	0,0017	Error 3> Error 4
TGP	0,0819	0,0227	0,0052	0,0791	0,0216	0,0080	Error 3< Error 4
SDP	0,0069	0,0147	0,0015	0,0076	0,0142	0,0021	Error 3< Error 4
MAX	0,0045	0,0100	0,0045	0,0058	0,0094	0,0057	Error 3< Error 4
DIG	0,0392	0,0171	0,0002	0,0396	0,0176	0,0006	Error 3< Error 4

Source: Authors' Own

5 Error 3 = real expected return - estimated expected return (for $\beta_{\text{Gini},i} = \beta_i$ and $k_{islamic} \coloneqq zakat$).
6 Error 4 = real expected return - estimated expected return (for $\beta_{\text{classic},i} = \beta_i$ and $k_{islamic} \coloneqq zakat$).

By referring to Table 7, we can ascertain that the Gini model outperforms the classical model when considering the scenario where $k_{islamic} := r_{inflation}$. This conclusion is supported by the fact that approximately 80% of the errors generated by the Gini model are lower than those generated by the classical model. However, it is worth noting that the Gini model, which incorporates zakāh as a substitute for the risk-free rate, is even more significant and relevant compared to the Gini model that employs inflation as a substitution. This finding highlights the superior effectiveness of incorporating the concept of zakāh within the Gini model, further emphasizing its suitability for capturing the unique dynamics of Islamic finance.

Table 7: Sharīʼah CAPM for $k_{islamic} := r_{inflation}$

	Model III: $\mathbb{E}[Y_i] - r_{\text{inflation}} = \alpha_{i,3} + (\mathbb{E}[X_m] - r_{\text{inflation}})\beta_{\text{Gini},i} + \epsilon_{i,3}$						
	$\beta_{\text{Gini},i} = \beta_i$			$\beta_{\text{classic},i} = \beta_i$			
Assets	Mean (%)	SD	Error 5[7]	Mean (%)	SD	Error 6[8]	Comparison
TNB	0,0272	0,0093	0,0018	0,0273	0,0095	0,0016	Error 5 >Error 6
PCG	0,0270	0,0177	0,0011	0,0278	0,0192	0,0018	Error 5<Error 6
PMB	0,1873	0,0259	0,0260	0,2006	0,0280	0,0393	Error 5 <Error 6
IHH	0,0317	0,0095	0,0045	0,0312	0,0091	0,0050	Error 5 <Error 6
AXIA	-0,0154	0,0217	0,0075	-0,0138	0,0206	0,0059	Error 5 >Error 6
DIGI	0,0141	0,0119	0,0003	0,0145	0,0108	0,0006	Error 5 <Error 6
TGP	0,0813	0,0227	0,0058	0,0782	0,0216	0,0089	Error 5 <Error 6
SDP	0,0064	0,0147	0,0010	0,0068	0,0142	0,0014	Error 5 <Error 6
MAX	0,0033	0,0100	0,0032	0,0041	0,0094	0,0041	Error 5 <Error 6
DIG	0,0393	0,0171	0,0003	0,0399	0,0176	0,0009	Error 5 <Error 6

Source: Authors' Own

7 Error 5 = real expected return - estimated expected return (for $\beta_{\text{Gini},i} = \beta_i$ and $k_{islamic} \coloneqq r_{\text{inflation}}$)
8 Error 6 = real expected return - estimated expected return (for $\beta_{\text{classic},i} = \beta_i$ and $k_{islamic} \coloneqq r_{\text{inflation}}$)

When examining the scenario where, $k_{islamic} = r_{NGDP}$, as depicted in Table 8, it becomes apparent that the Gini model exhibits significantly smaller errors compared to the classical model. In fact, approximately 90% of the errors associated with the Gini model are smaller than those of the classical model. Consequently, based on this compelling evidence, we can confidently conclude that the Gini model surpasses the classical model in terms of accuracy and reliability. As a result, the Gini model is deemed preferable over the classical model for our analysis.

Table 8: Sharīʼah CAPM for $k_{islamic} = r_{NGDP}$

	Model IV: $\mathbb{E}[Y_i] - r_{NGDP} = \alpha_{i,4} + (\mathbb{E}[X_m] - r_{NGDP})\beta_{Gini,i} + \epsilon_{i,4}$						
	$\beta_{\text{Gini},i} = \beta_i$			$\beta_{\text{classic},i} = \beta_i$			
Assets	Mean (%)	SD	Error 7[9]	Mean (%)	SD	Error 8[10]	Comparison
TNB	0,0303	0,0093	0,0013	0,0302	0,0095	0,0012	Error 7> Error 8
PCG	0,0245	0,0177	0,0015	0,0234	0,0192	0,0026	Error 7< Error 8
PMB	0,1838	0,0259	0,0225	0,1953	0,0280	0,0341	Error 7< Error 8
IHH	0,0364	0,0095	0,0002	0,0365	0,0091	0,0003	Error 7< Error 8
AXIA	-0,0210	0,0217	0,0131	-0,0182	0,0206	0,0102	Error 7< Error 8
DIGI	0,0153	0,0119	0,0015	0,0173	0,0108	0,0035	Error 7 < Error 8
TGP	0,0829	0,0227	0,0042	0,0806	0,0216	0,0064	Error 7 < Error 8
SDP	0,0079	0,0147	0,0024	0,0089	0,0142	0,0034	Error 7 < Error 8
MAX	0,0067	0,0100	0,0066	0,0086	0,0094	0,0085	Error 7 < Error 8
DIG	0,0390	0,0171	0,0000	0,0391	0,0176	0,0001	Error 7 < Error 8

Source: Authors' Own

9 Error 7 = real expected return - estimated expected return (for $\beta_{\text{Gini},i} = \beta_i$ and $k_{islamic} \coloneqq r_{\text{NGDP}}$)
10 Error 8 = real expected return - estimated expected return (for $\beta_{\text{classic},i} = \beta_i$ and $k_{islamic} \coloneqq r_{\text{NGDP}}$)

Remark 5. In relation to risk, specifically standard deviation (SD), it is observed that certain assets are overestimated in terms of risk, while others are underestimated. However, when it comes to precision, the models provided by Mean Gini exhibit better performance. This statement implies that while there may be discrepancies in risk estimation for different assets, the Mean Gini models excel in terms of accuracy and precision. It suggests that the Mean Gini approach provides more reliable and precise risk assessments compared to other models.

The outcomes of our comparative analysis lead us to conclude that the Gini models outperform the classical models in terms of efficiency. In particular, we can confidently assert that the Gini model, where the risk-free rate is replaced by the concept of "zakāh," proves to be the most suitable choice. This assertion is supported by the fact that 90% of errors 1 (Model II) are smaller in magnitude compared to errors 7 (Model IV). Thus, the Gini model with the inclusion of zakāh demonstrates superior performance and can be considered as the preferred option.

6. Conclusion

In recent two decades, there has been a growing global confidence in the Islamic economic and financial system, which is in line with the advancements in research and the expanding domain of Sharī'ah finance in the investment world. However, the existing literature on Sharī'ah modelling for portfolios is still insufficient. The presence of the Sharī'ah system in the investment landscape is anticipated to cater to the needs of Muslim investors and provide them with suitable opportunities. As a result, there is a pressing need for further research and development in the field of Islamic finance to meet the growing demand and enhance understanding in this area. The emergence of Islamic finance has sparked our inspiration to seek an alternative to conventional asset pricing models. Our goal is to construct a more robust model that can effectively analyze the risk and the relationship between return and risk in Sharī'ah- compliant investments.

In this paper, we begin by presenting supportive evidence from the literature that establishes the significance of using the Capital Asset Pricing Model based on Mean Gini (MG) in contrast to CAPM based on Mean Variance (MV). Building upon this foundation, we introduce the incorporation of MG in portfolio optimization within the Islamic framework. This involves eliminating the risk-free rate and integrating new parameters, some of them are derived from Sharī'ah principles, namely purification and zakāh. Our objective was to demonstrate the optimal Beta based on the Gini coefficient, thus presenting the Sharī'ah CAPM (MG-SCAPM) for Islamic investors. Continuing with our exploration, drawing inspiration from the existing literature, we propose five models for pricing Islamic assets in Section 3. These models expand the understanding and analysis of Islamic finance, offering valuable insights into the pricing dynamics specific to the Islamic financial system. Moreover, we selected ten prominent Islamic constituents from the Malaysian Stock Exchange to perform a real data analysis. Based on our findings, we can deduce that Model II, represented by the equation: $\mathbb{E}[Y_i] - z_i = \alpha_{i,2} + (\mathbb{E}[X_m] - z_i)\beta_{\text{Gini},i} + \epsilon_{i,2}$, demonstrates the highest level of optimality compared to the other models. This conclusion is supported by the fact that Model II exhibits the fewest errors when compared to the alternative models under consideration. Our research highlights the effectiveness of Model II in capturing the underlying dynamics and providing accurate predictions within the given context.

In the theoretical framework, we have introduced Models V and VI, which have not been empirically tested in practical applications. This highlights the need for future research to explore these models across various stock markets, taking into account factors such as the Sukuk rate and the market purification rate. Such studies hold great potential for advancing the field of Islamic finance by providing deeper insights into the relationship between risk and return. As Islamic finance is anticipated to become a prominent financial sector, the investigation of these models can significantly benefit the industry and contribute to its overall understanding and development.

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تسعير الأصول الإسلامية باعتماد متوسط جيني

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المستخلص. في سياق الأهمية الاقتصادية التي اكتسبها الاستثمار الخُلقي، استحوذت المالية الإسلامية على أهمية متزايدة كخيار استثماري، ولم يقتصر ذلك على الدول ذات الأغلبية المسلمة فحسب؛ بل امتد ليشمل أيضًا الكثير من الدول الغربية. في هذا الإطار، قدم الباحثون في هذه الورقة نموذجًا لتسعير الأصول التي يمكن اعتبارها متوافقة مع الشريعة الإسلامية إلى حد كبير، وهو "نموذج متوسِّط جيني لتسعير الأصول المُتوافقة مع الشّريعة الإسلاميّة ". يتضمن النموذج المقترح متغيرات يمكن اعتبارها أقرب إلى قواعد وأصول المالية الإسلامية؛ متغيرات كالزكاة، ومعدل التضخم، ومعدل النمو الاقتصادي، وعائد الصّكوك، كبدائل لمعدّل خالٍ من المخاطر (سعر الفائدة). علاوة على ذلك، قمنا بإجراء دراسة تطبيقيّة على أصول من بورصة ماليزيا بهدف توفير نموذج أكثر ملاءمة للمستثمرين المُسلمين، وتبيّن أن النّموذج الذي يستبدل معدّل الفائدة بالزّكاة هو الأكثر ملاءمة للأصول التي تمّ اختيارها.

الكلمات الدَّالة: المالية الإسلامية، الشريعة، متوسط جيني، تحسين المحفظة الاستثمار، النموذج التقليدي لتسعير الأصول.

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